Why Teach Mathematics?

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Why teach mathematics? What is the purpose of an enterprise which, in the elementary schools of the USA, involves some ten billion child-hours each school year and which costs American taxpayers and parents some thirty million dollars a year in textbooks, to say nothing of materials, teachers' salaries, or money spent for testing, teacher preparation, or curriculum development?

Why teach mathematics? So that children can multiply a two-digit number by a three-digit number? So that children can say "parabola" when passing a McDonald's? So that children can say the word "commutativity" when "8 + 7 = 7 + 8" is written on the board? These are mini-objectives. They are far too specific for the discussion at hand.

Why teach mathematics? To make new mathematicians? To "train minds"? To enable children to use some standard computational algorithms? To teach children to "think"? To help children understand certain principles of various number systems? To enable children to reproduce, on request, certain facts, rules, and procedures of arithmetic?

The goals of present elementary-school mathematics programs—derived largely from the publications of the School Mathematics Study Group—

involve an understanding of the principles of the rational number system and the interrelatedness of various subsets of the rationals, the use of underlying principles such as associativity and distributivity in support of proficiency in computation and, to a lesser extent, a knowledge of nomenclature and notation in geometry.

Some questions must be raised about present textbooks and about teachers who follow these textbooks without constructing for themselves a position on the issue "Why teach mathematics?" Why should there be such a great emphasis on number and geometry? In these areas, what is the usefulness in the life of a child—or in the life of the adult he will become—of an understanding of the structure of the rational number system? What is the usefulness of a "knowledge" of facts, rules, and procedures of number? What is the marketability of computational skill when thousands of error-free long divisions, for example, can be performed by a computer in a few seconds at a cost of a few cents? What is the usefulness in the life of a child or an adult of a "knowledge" of the nomenclature and the notation of elementary geometry? If problem-solving is limited to verbal exercises like "John had 3 cats and Mary had 4 cats . . .," has problem-solving as it is understood by mathematicians and psychologists really been given the attention it deserves?

Doubtless there are some good answers to some of these questions. For example, proficiency in computation has some usefulness to children who want to know how many days to Christmas or to adults who need to estimate the cost of a dinner for their family. But is the attention that mathematics education gives to proficiency in computation justified? Could the time spent on developing proficiency in computation be more profitably expended elsewhere? What are the global goals of elementary-school mathematics?

The purpose of these questions is to ask teachers to construct for themselves an answer to the question "Why teach mathematics?" If this article does nothing more than to focus teachers' attention on such a question, it will have succeeded, for teachers tend too easily to accept the role of conduit of "knowledge" between textbooks and children without asking questions about the goals that "knowledge" purports to serve. Such questions must be asked.

The purpose of this article is to suggest the beginning of an answer to the question "Why teach mathematics?"—an answer that differs from one suggested by today's elementary-school textbooks and classrooms.

In a not-so-recent but farsighted article in *The Mathematics Teacher,* Carl B. Allendoerfer, former president of the Mathematical Association of America, called for a second revolution in school mathematics (1). "In spite of all the thunder and lightning [of post-Sputnik developments in mathematics education]" he wrote, "I believe that we have not made any fundamental change in school mathematics. We have taken a structure, which was well built to start with, have shifted its partitions, redecorated its rooms, put in new and faster elevators, and cleared the grime of the exterior . . ." (1: 692). The post-Sputnik movement, said Allendoerfer, "was guided by college people and was largely aimed at the college-capable students. The construction of the curriculum and the preparation of the text materials were not based on any theory of learning or educational research; they represent solely what the writers thought to be best in the light of their knowledge of mathematics and their experience in the classroom" (1: 692).

Allendoerfer then called for the construction of a "drastically different" curriculum in mathematics, one that attends to early grades first in contrast to post-Sputnik developments and that "pays attention to the growing body of knowledge on how young children learn mathematics" (1: 693). Such a curriculum would involve the "construction, with the help of psychologists, neurosurgeons, and others, of a drastically different elementary curriculum in mathematics adjusted to the learning patterns of young children" (1: 695).

Indeed, if there was one failing in the post-Sputnik movement, it was that almost no attention was given to children's thinking and learning. Rather, as Allendoerfer suggested, curriculum-makers looked to mathematics as a body of content and to the interface of mathematics with societal and technological needs, but they ignored what was known—by teachers and researchers—about young children and their interests, abilities, thinking, and ways of learning (2). Knowledge of children's thinking and learning must be taken into account if we are to be more successful in bringing about mathematics learning by children. But this knowledge can do more than merely help us to do old things in better ways. It can help us to define new goals for school mathematics.

New goals spring naturally from the research on children's thinking that Jean Piaget has amassed over the past fifty years. The major burden of Piagetian research has been to describe the evolution of thought from infancy to adulthood, and one central finding is that children's thinking is different from the thinking of adults.

Piaget has described the development of thinking in terms far too technical and detailed to be described adequately here. The general reader can obtain detailed though largely secondary background on Piagetian research on children's thinking in sources listed at the end of this article (3–8). In brief, the period of concrete operations—which coincides roughly with the elementary-school years—is the time for the construction of thought operations involving classes and relations by which children can deal with two or more judgments simultaneously. These operations enable the child to recognize the invariance of certain attributes in the face of certain kinds of transformations—for example, length through displacement and numerousness through spatial rearrangement. In short, concrete operational thinking involves the triumph of concept over percept, decentration—whereby children focus on and correlate more than one aspect of a situation at a time—and reversibility, which enables children to retrace the steps of an argument and to proceed back and forth from one state to another over a transformation.

One major characteristic of children's thinking in the elementary-school years—the period in which concrete operations are constructed—is atomism. Atomism is the child's view that the things, the events, and the ideas that he experiences are unrelated to one another. For the child, they exist as isolated atoms. (The author's use of *atomism* in this article is quite different from Piaget's (9). By *atomism* Piaget means a view of things as made up of tiny grains. I have used *atomism* in a completely different way in the hope that teachers will find the word useful in referring to the characteristics of children's thinking.) Teachers are likely to be acquainted with the examples of atomism listed here:

1. Hold up five fingers. Child counts: "One, two, three, four, five." Hold up the same five fingers and one finger of the other hand. Child counts: "One, two, . . ." The child does not see the relation between five and six. Five is one issue, six is another.

2. Young children are not able to understand that a person can reside in Chicago and Illinois at the same time or that a person can be a teacher and a mother simultaneously. In fact, even some high-school students are surprised to see their teachers shopping for groceries or taking out the garbage.

3. Study a child's behavior in playing tick-tack-toe. Young children are often incapable of concentrating on offense and defense simultaneously. In tick-tack-toe, for example, a child with X will play as shown here, ignoring the chance for a sure kill.

Original situation

X		
O		X
	O	

Child plays

X	X	
O		X
	O	

Rather than

X		X
O		X
	O	

4. Play "Twenty Questions" where the universe is people. Children's behavior is likely to be atomistic in that their questions are unrelated to one another or are inconsistent with previously given information. Further, children often will not use questions involving classifying; they will ask "Is it Jack?" or "Is it Grandma?" rather than "Is it a male?" or "Is it an adult?"

5. Play the game "I Am Thinking of a Number" where the numbers run from 1 to 100 and children are allowed only questions of the type "Is it greater than . . . ?" and "Is it less than . . . ?" The same instances of atomism are likely to occur. Further, many children are incapable of transitivity. Having established that the mystery number is less than 40, someone is almost certain to ask, "Is it less than 50?"

6. Ask children to define a word such as "mother." They will not attend to the necessary and sufficient conditions of class membership. Rather, they will focus on the perceptual characteristics of the object or on its role in their lives. One marvelous response to the question "What is a mother?" heard recently was "A person with those lady things on her chest." For other examples of atomism, see Richard W. Copeland's book (10).

Children also exhibit what has been called "backwards atomism"—the imposition of relations between atoms without sufficient testing as to whether the relations apply.

7. After the flag on a rural mailbox was put up to post letters on several consecutive days on which mail also arrived, an eight-year-old said, "We'd better put up the flag today." "Why?" "So that we'll get some mail."

8. Given the fact that appendicitis results in pain in the lower right abdo-

men, children (adults, too?) jump to the conclusion that such a pain necessarily indicates appendicitis.

It is conceivable that elementary-school mathematics curriculums reinforce, or at least perpetuate, the natural atomism of children's thinking. It is not difficult to imagine that transmission, drill, and the memorization of facts, rules, procedures, nomenclature, and notation of elementary arithmetic and geometry encourage the view that the world—and especially the part of the world called *mathematics*—consists of events, percepts, and actions that are arbitrary and unrelated. At the very least, given the widely documented cognitive differences between children and adults, it is reasonable to ask whether a mathematics curriculum that is coherent to adult mathematicians is necessarily coherent to children of elementary-school age.

One major global objective of education in the elementary school should be to overcome atomism in children, that is, to build relational thinking in children. *Relational thinking* is thinking that weaves together the atoms of a person's experience. When a person classifies, for example, he is thinking relationally. When a person orders atoms, he is thinking relationally. When he weaves together atoms to produce a conclusion that has no alternative, he is thinking relationally. The building of a generalized logic—called here *relational thinking*—is the major goal of education (11). Charlesworth writes:

> Most of the cognitive structures elaborated upon by Piaget share similar beginnings—their first forms develop as a result of the child's prolonged everyday contact with the physical phenomena. . . . Toys, furniture, and people preserve themselves even while hidden from view; the substance, weight, and volume of candy, pencils, and water are always conserved despite changes in their form or distribution; arithmetical operations, which Piaget conceives of as derivatives of physical actions upon physical objects, perform their functions with amazing constancy and accuracy; and logical propositions are bound together in wonderfully immutable relationships. And as a further consequence of all this, the child gradually learns what is possible and what is impossible and also the distinction between the two. He develops a sense of necessity that certain events must occur, that others can never occur, and that certain relationships (e.g. transitivity) between objects or representations of them must hold forever. . . . The organization and coordination of what he knows into a gradually tightening logical system provides him with a powerful method of reducing a sizable segment of reality to predictable and hence controllable proportions [12].

To what extent do present-day elementary-school mathematics curriculums play a role in this development?

That the building of relational thinking should be a major goal of mathematics education is not immediately apparent. Mathematics, it is commonly believed by elementary-school teachers, is concerned chiefly with algorithms, rules, procedures, conventions, and notation. But mathematics, in the view of the Bourbaki group, an anonymous assemblage of leading mathematicians, is the study of structures, the study of systematic patterns and relationships. Piaget's work itself closely allies human thought with mathematical structures. Perhaps it is reasonable, then, to suggest that the building of relational thinking is precisely the concern of mathematics educators and of teachers of mathematics in the elementary school.

There are likely to be objections to the proposal that the building of relational thinking should be a major goal of mathematics education for young children. One objection is that things are going along quite well as they are, that the present curriculum, which deals with principles and procedures of adult grown-up mathematics, serves well to build cognitive structure in children. This may very well be true, though it cannot be doubted that a more direct attack would be likely to have greater payoff. Perhaps teachers can bring their own evidence to bear on the question. It is the author's impression that our present curriculums have little effect on children's relational thinking. What we have succeeded in doing—more or less—is to get children to give correct answers to issues that are trivial from the standpoint of developing children's thinking. Because we have been concerned with products of children's thinking rather than with the underlying processes behind the products, we have too often accepted what Piaget calls "false accommodation"—strings of words unsupported by stable or generalizable thinking—in place of relational thinking. In short, we have looked too often at memory when we should have looked at mind.

There is some research evidence that this is so. Lovell (13) cites Van Engen and Steffe, who found that of one hundred first-grade pupils:

> . . . almost all could correctly work using symbols, 2 + 3 = and 4 + 5 =. Yet only 54 correctly stated no preference for separate or combined piles of candies when 5 were used, and only 45 when 9 candies were employed. It would appear that many pupils in the first grade can memorize facts employing symbols, but are unable to abstract the concept of addition from physical situations. Likewise Steffe showed that only 61 out of 341 first-grade pupils conserved numerousness over 4 items in each of three tasks when the numbers did not exceed 8, and that 128 pupils responded incorrectly to at least one item of the 4 in each of three tasks. But on a test of addition facts with totals not greater than 8, pupils of the second group had an average score of 76%, and pupils of the first group an average score of 91%. These data again warn us that addition facts can be learned without a firm abstraction of number [13: 4].

A second objection to relational thinking as a goal of mathematics education is that such thinking develops naturally—without intervention—and that attempts to influence cognition are futile. This position is often attributed to Piaget himself. In fact, nothing could be further from the truth. Maturation is only one of four factors contributing to cognitive development, says Piaget. The other three—social transmission, physical and logico-mathematical experience, and equilibration—are susceptible to systematic influence such as would take place in an educational setting.

There is some evidence, at least at the higher levels of cognitive development, that random influences alone do not do the job. McKinnon and Renner report that Friot showed that 82 per cent of eighth- and ninth-grade children were still at the stage of concrete operations. Further, in McKinnon's own research, 50 per cent of university Freshmen were found to be at a concrete operational level of thinking and another 24 per cent were in a stage of transition from concrete operational thinking to formal operational thinking (14). Further evidence from a long series of studies suggests that the ability to deal formally with inference patterns involving "if . . . then" statements develops very slowly and that students of high-school age are often not able to distinguish between a valid and an invalid argument (15). Perhaps the most convincing argument that maturation is not a sufficient precipitator of cognitive development is the work of Pinard, who found that children in Martinique achieve conservation of numerousness some four years later than Swiss children do (6: 169).

If maturation and random experience do not always carry children through to formal operations, must we be content to let natural influences—whatever they are—carry the day? Is intervention a futile enterprise? Not at all. Piaget himself writes, "In an article written recently for the *Encyclopaedia Britannica,* R. M. Hutchins affirms that the principal aim of education is to develop the intelligence itself, and above all to teach how to develop it 'for as long as it is capable of further progress,' which is to say, of course, far beyond the age at which one leaves school. . . . But it is also quite clear that this formula does not mean very much unless one can be quite precise about what intelligence consists of . . . Fortunately, however, it is precisely in this field [characterising intelligence according to its modes of formation and development] that child psychology has provided us with most new results since 1935" (16: 27). For Piaget, intelligence "consists in executing and coordinating actions, though in an interiorized and reflective form. These interiorized actions . . . are nothing other than the logical or mathematical 'operations' that are the motors of all judgment, or reasoning" (16: 29). For Piaget, then, the goal of education is logical coherence.

At one extreme is the position that maturation is the only influence in the development of logical coherence. At the other is the view that the intellectual development of the child is due to experience alone. Coordinating the two positions is the Piagetian view:

> Although we cannot at present fix with any certainty the boundary between the contribution of the mind's structural maturation and that of the child's individual experience or the influences exerted by his physical and social environment, it does nevertheless seem that we should accept both that these two factors are constantly at work and that development is a product of their continuous interaction. From the point of view of schooling, this means, in the first place, that we must recognize the existence of a process of mental development; that all intellectual raw material is not invariably assimilable at all ages; that we should take into account the particular interests and needs of each stage. It also means, in the second place, that environment can play a decisive role in the development of the mind; that the thought contents of the stages and the ages at which they occur are not immutably fixed . . . [16: 172–73].

Does the building of relational thinking as a major goal of mathematics education mean that we should accelerate children's cognitive development? It is hard to answer this question because "accelerate" suggests that Piagetian research has identified some hard and fast schedule for cognitive development which cannot be altered. This is not at all true. What is suggested here is that educational approaches can be devised that give children the experience necessary for particular developments to occur: without these experiences the developments may never occur. It is a major role of the school to provide in a concise and systematic way certain experiences that would otherwise occur randomly (or not at all) in a child's everyday life.

Finally, does the suggestion that relational thinking be the major goal of elementary-school mathematics imply that mathematics, as we know it, should be a thing of the past? Yes and no. What is suggested is that the early part of a child's mathematical experiences consist of activities that match with and extend his thinking rather than activities that call merely for the storage and the retrieval of atoms, be they facts or procedures. Does such an approach mean that children will never learn and apply the body of content called *mathematics* and that they will never generate new mathematics? Certainly not. It means that children will start in a different way, a way that calls for the constructing of primitive logical ideas so that children can be agents, not patients, in mathematizing. As Polya has often said, "Mathematics is not a spectator sport" (17). While a different starting place is called for, the ending need not be different in content, save that pupils are likely to be more able in their achievement—*construction* is a better word—of mathematical ideas, principles, and procedures.

There are at least two dangers that must be kept in mind in any attempt to influence the cognitive structure of children. One danger is that teachers will settle on a precise and limited attempt to influence success on specific Piagetian tasks. This is a futile enterprise. The tasks are simply indicators of children's underlying thinking. Head-on attempts to get children to "pass" a task, however successful they may be, are likely to be futile in stability and generalizability.

A second danger is perhaps more important. It is unlikely that children can be "taught" to think relationally. It is unlikely that teachers can transmit relational thinking to children as one might get a person to memorize a telephone number. Children need to construct relational thinking for themselves in the face of problem situations that involve moderate dissonance. What moderate dissonance does in such situations is to set in action the equilibration mechanism that results in accommodative modification of the children's original point of view.

Piaget, in summarizing "the main conclusions that the varied researches of child psychology have offered pedagogy in the last few years," writes:

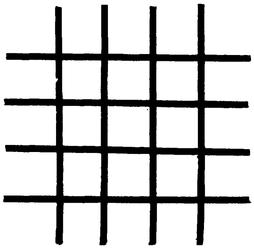
> On the one hand, this [intellectual] development is essentially dependent upon the activities of the subject, and its constant mainspring, from pure sensorimotor activity to the most completely interiorized operations, is an irreducible and spontaneous operativity. On the other hand, this operativity is neither preformed once and for all nor explicable solely by the external contributions of experiment or social transmission: it is the product of successive constructions, and the principal factor in this constructivism is an equilibration achieved by autoregulations that make it possible to remedy momentary incoherences, to resolve problems, and to surmount crises or periods of imbalance by a constant elaboration of fresh structures that the school can ei-

ther ignore or encourage according to the methods it employs [16: 40-41].

The activities in the following list are likely to be useful in developing relational thinking in children. The activities are presented as examples of the problem situations just described. Further, they should help the reader construct for himself a definition of "relational thinking" in trying the tasks with children and observing their responses and making inferences about their thinking. Finally, they provide a beginning (and sketchy) set of problems that teachers might use—and improve and extend as they learn from children—to help children build relational thinking. No age guidelines are provided for teachers. To find a good fit with their pupils, teachers will have to look carefully at children's thinking and, in the light of the evidence they obtain, modify the activities.

1. Classification:

Have children make a grid like that shown here and have them make five red, five green, five blue, five white, and five orange paper squares.



Ask the children to use a 2 by 2 section of the grid and arrange the blues and the whites so that there is no duplication of color in any row column, row, or main diagonal. Can't do it? Is it logically impossible? Why? How many colors would it take?

Ask the children to try for a 3 by 3 with three blues, three whites, three greens.

Now a 4 by 4 with four chips in each of four colors. How many different 4 by 4's are possible?

Is a 5 by 5 possible? How many of them?

Generalize.

2. Classification, Logical Necessity, Transitivity:

I'm thinking of a number from 1 to 100. Try to guess the number. You can ask questions of the type "Is it less than 20?" or "Is it more than 75?"

I'm thinking of two numbers from 1 to 100. Same rules as above. If your condition ("Less than 42?") holds for both numbers, I'll answer "Yes." Otherwise, I'll answer "No."

I'm thinking of two numbers from 1 to 100. If your condition holds for one number, I'll answer "Yes." Otherwise, "No."

I'm thinking of two numbers from 1 to 100. If your condition holds for precisely one number, I'll answer "Yes." Otherwise, "No."

3. Logical Necessity:

Two children get a complete, shuffled deck of cards.

One child selects a card at random. The other child has to determine which card was taken.

I select a card at random. Then you choose cards at random and I'll tell you in how many ways (value and suit or value, suit, and color) each card is different from the unknown card. Your task is to identify the unknown card. (This activity can also be undertaken with Attribute Blocks.)

4. Logical Multiplication: Pegity (18).

Similar to tick-tack-toe, except that it is played on a 10 by 10 grid. The winner is the first player to capture five contiguous boxes horizontally, vertically, or diagonally.

5. Combinatorial Thinking:

You and I write the letters of our first names on as many paper squares as needed. Then we mix the squares, select a random square, and then take four squares each. You and I make as many English words as we can using at least one of the four chips and the randomly drawn chip.

It is all too easy to think that a brief set of activities such as these will cause children to construct relational thinking. These activities are simply examples of equilibration-provoking perturbations which, one hopes, might be effective in inducing relational thinking. Two points must be made. First, it is not at all clear—given the present state of the art—what effect various perturbations might have on children's relational thinking. The field is wide open for research; and these activities, in particular, demand research. Second, a small set of activities is by no means likely to induce relational thinking. What is needed, in Allendoerfer's words, is a "drastically different curriculum in elementary mathematics, one which starts from the bottom up and which pays attention to the growing body of knowledge on how young children learn mathematics" (1: 693).

It is clear from the work of Dienes (19), Sinclair (20), and Biggs (21), to say nothing of the work of The Elementary Science Study (22) and The Cambridge Conference (23), that we have made a start. It is also clear, to this observer at least, that we have a long way to go and a great deal of work to do.

The goal of mathematics education in the elementary school should not be limited to a knowledge of the facts, rules, procedures, nomenclature, and notation of elementary-school arithmetic and geometry. Such an approach may serve to perpetuate, if not to increase, the natural atomism of children. The goal of elementary-school mathematics education should be the building of relational thinking by children. The job of building a systematic approach in support of such a goal is a massive and difficult one, but it needs to be tackled by teachers, curriculum developers, and researchers.

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