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SOME NOTES ON MULTIPLICATION OF WHOLE NUMBERS

Following are some brief comments on an approach to multiplication of whole numbers. The approach, while not new, has not been widely used with young children, and it is a possibility that the development suggested herein may offer some advantages over the various approaches in current use.

In the present approach, multiplication of whole numbers is seen in terms of a street-crossing model. If there are 4 ↔ streets and 3 ↕ streets, there are 4×3 intersections or crossings.

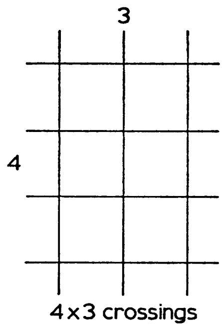

Fig. 1

In general, for a ↔ streets and b ↕ streets, there are $a \times b$ crossings.

Suppose, for example, that four boys each get three cookies, or suppose that a tap releasing water at three gallons per minute runs for four minutes. The model is essentially the same:

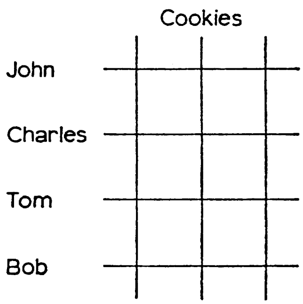

Fig. 2

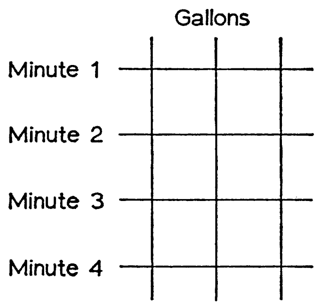

Fig. 3

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In each case, the grid model allows children to see whole-number multiplication for what it is – *counting* the elements when there are *a* sets of *b* elements. The grid model is a generalized form of the commonly used array model; its advantage is that it leads to a straightforward algorithm which is easily devised by students (see below).

An alternate approach is to use regions rather than crossings:

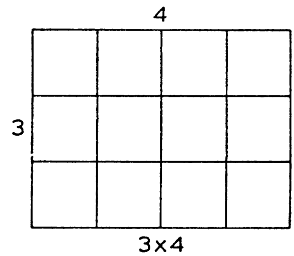

Fig. 4

While this approach has in fact been used to explicate whole-number multiplication, the viewpoint of this writer is that it obscures the nature of the issue – counting. Further, it seems unrealistic to expect young children to draw such a cumbersome picture and thus to have easy and constant recourse to that model for whole-number multiplication.

Suppose we agree to use the street-crossing model for basic multiplication facts. At any time a child can draw the street-crossing model and count to get an answer. The next task then becomes one of extending from basic facts to the computation of products such as 34×27, 357×21, and so forth. Traditionally, we have approached this issue in an atomistic way. First we have allowed children to deal with 1-digit by 1-digit products, then 2-digit by 1-digit, 3-digit by 1-digit, 2-digit by 2-digit, and so forth. The present approach does not fragment the issue; rather, it simply allows application of the street-crossing model to any instance of the general case of $a \times b$, where *a* and *b* are whole numbers (Figure 5).*

George Polya says that mathematics is "being lazy". That is, mathematics means letting principles and generalizations do the work for us. Since counting the crossings for 34×27 is a tedious task, we might try to look for some labor-saving mathematical principles. One approach involves bridging, i.e., extend basic facts such as 2×3 to 2×30, 20×30, 200×3, and so forth.

* Graph paper has been used here for ease of production in this journal. In classroom practice, students would draw their own streets.

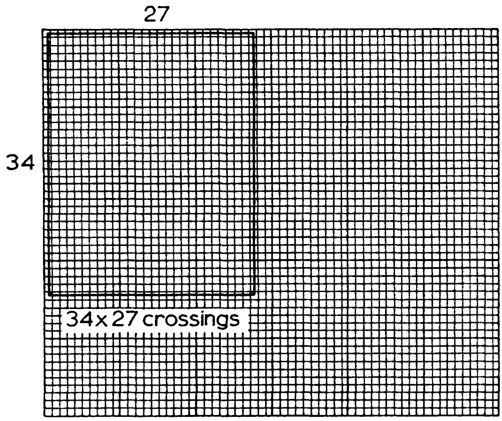

Fig. 5

(1) Suppose the issue is 30 × 20.
(2) 30 is 3 tens, 20 is 2 tens.
(3) 3 tens × 2 tens = 6 hundreds (Figure 6).

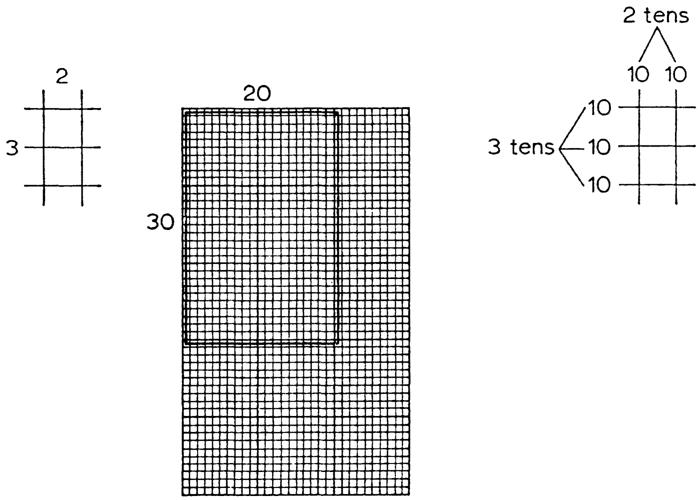

Fig. 6

In general, m tens × n tens = $(m \times n)$ hundreds. Similar arguments can be established for other powers of ten in order to bridge from basic combinations.

Now we can employ one of Polya's heuristics for problem-solving. We can break up the original problem into smaller problems. Using basic facts and bridging from basic facts, we can break up 34 × 27 into 4 simple problems (Figure 7).

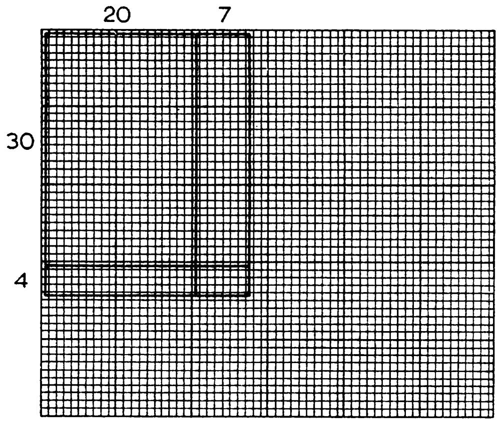

Fig. 7

The next step is to collapse the argument to a simpler form, still preserving the grid-crossing model:

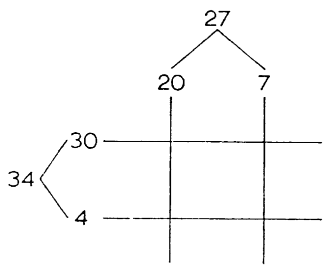

Fig. 8

And, if one wishes, he can record the computed partial products right on the grid:

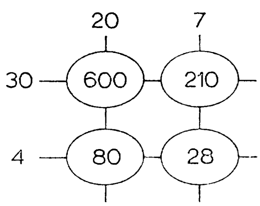

Fig. 9

Similarly, for the computation of other products, the same model holds:

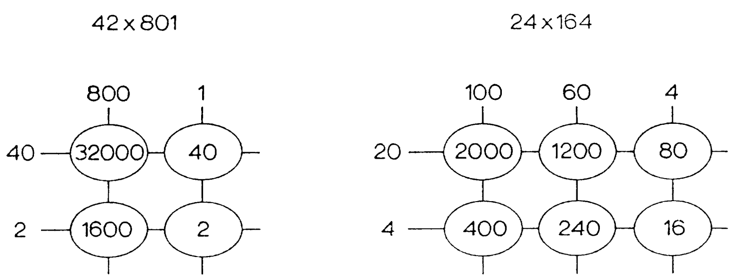

Fig. 10

The arguments in favor of the street-crossing model are several. First, the idea is a simple one and the model is easily constructed by children. Second, the street-crossing model subsumes various other approaches. Want to view whole-number multiplication as repeated addition or in terms of Cartesian product? The model allows for such interpretations. Third, the street-crossing model ignores the traditional and trivial classification system which emphasizes differences between n-digit $\times$ m-digit multiplication exercises and substitutes in its place a simple and pervasive concept of whole-number multiplication. Fourth, the approach allows for meaningful employment of the powerful distributive principle without an elaborate fuss of an axiomatic nature, viz.:

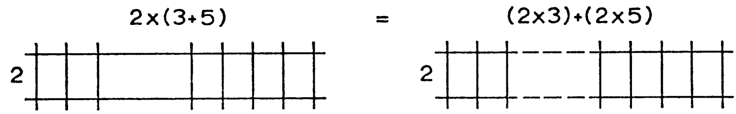

Fig. 11

With the distributive principle so apparent, the stage is set for the use of the street-crossing model in division of whole numbers. Fifth and finally, the model gets at the heart of multiplication: for each member of Set A there are b members in Set B. Since there are only two arithmetic situations – additive and multiplicative – the street-crossing model is likely to make it easier for the child whose task it is to distinguish whether a real-life number situation is additive or multiplicative.

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