Fasten Your Seat Belts

A teacher and her mentor invite readers along for the ride as they use a fun logic game to introduce sixth-graders to advanced mathematical thinking. Mr. O'Brien and Ms. Barnett discover that this new route allows even students who previously fared poorly in math to arrive at the destination with their peers.

BY THOMAS C. O'BRIEN AND JUDY A. BARNETT

WHEN IT CAME time to put a title on this article, we called it "Fasten Your Seat Belts" for two reasons. First, in the classroom tryouts of these activities, we had no idea what was going to happen next. We were entering uncharted territory for us, both veteran teachers, and for the children. And second, we hope readers will want to try the activities for themselves rather than merely read about them. And in that case, readers are advised to fasten their seat belts.

One of the major characteristics of mathematics is that one can *derive* new information from old information with logical certainty. Put a penny on one hand, make a fist with both hands, show the fists to a child of 5 or so, and ask the child to find the penny. If the child chooses the fist with the penny, she will see it when the fist is opened. If the child chooses the other hand and is shown an open hand without a penny, she can't see the penny in the unchosen hand with her eyes, but she can *see* the penny with her *mind*.

The fancy word for deriving information is infer-

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ence. Try the hide-a-penny task with a 2-year-old. No go. Two-year-olds are pre-inferential.

Inference is the heart of the mathematical concept of proof. The elementary and secondary school years are the time for the construction of logical operations and the development of tactics and skills that are critical for mature mathematical thinking. Indeed, such skills are central to thinking in everyday life.

THE BUNNY GAME

At odd intervals once a week over a period of several months, we conducted inference activities with several classes of sixth-graders in a midwestern American elementary school. Our starting point was an inference game that we called The Bunny Game.[1] What follows is a series of episodes from those classrooms, along with our comments on the thinking we observed. We alternated in the role of the teacher.

Teacher: Imagine that a Bunny is hiding in one of the boxes of a 4 x 4 grid. Your job is to find it. The rules of our game are simple. You can ask about various boxes. If the box you ask about is touching the Bunny sidewise (that is, on any of its four sides), you'll get the answer "Hot." Otherwise, the answer will be "Cold." For example, if the Bunny is in the box shown, and you ask about box B-1, the box is Hot because it touches the Bunny's box sidewise.

GAME 1

	A	B	C	D
1		H		
2				
3				
4				

Teacher: Can you show me another box that's Hot? [The children cite A-2.] And what about C-2? [The children say, "Cold."] And D-4? [Again the children say, "Cold."]

Children: What about the box the Bunny is in? Is it Hot or Cold?

Teacher: If you select the Bunny's box, you are a winner, and the game is over.

Working in friendship groups, the children played the game with the teacher selecting the Bunny's box at random. The children also played the game with one another while we functioned as roving observers. Quickly the children became very proficient, and we introduced "Bet your bicycle" as an expression of absolute certainty. For example, in Game 2, most children were willing to say they knew for sure where the Bunny was.

GAME 2

	A	B	C	D
1			H	
2		C		
3				
4				

Walter: The Bunny is in D-1.

Teacher: Are you willing to bet your bicycle?

Walter: Yes, and my dog and my little sister.

After a short period, we changed the rules slightly. To emphasize the act of inferring, we told the children that the Bunny game would now have two phases: gathering information and making a claim. Children could gather evidence as long as they wanted, but when they had enough information to tell with certainty where the Bunny was, they should say, "I am ready to make a claim." If their claim was wrong, they would be out of that round of the game.

Matthew: Okay. In the earlier game we could run across the Bunny by accident. Now what message

Illustration by Jem Sullivan

do we get if we ask about the Bunny's box when we are gathering information?

Teacher: You will be told Cold because the Bunny's box is not sidewise touching itself.

As the children played the game and grew more efficient, we challenged them to explain why this or that question was a good one. We commonly asked the children to take a minute or two to decide on the best possible question. And occasionally we would halt the questioning and ask the children to put an X in all the boxes that had been "zapped" by the available data.

Here are two situations that arose. (See Games 3 and 4.) In which case would you be willing to bet your bicycle? If you are not ready to make a claim, what box would be best to ask about next? (Answers to games appear on page 206.) A question that we thought about afterward but didn't ask was, "In general, how many boxes do you need to ask about before you can make a claim?"

GAME 3

	A	B	C	D
1	C			
2		H		
3				
4				

GAME 4

	A	B	C	D
1				
2	H		C	
3		C		
4				

After about an hour of these activities, most of the children (including some whose high level of participation surprised the teacher) were quite efficient at playing the game, and they asked about finding two Bunnies.

The rules for The Two Bunny Game are the same as the earlier ones except for the case in which the children ask about a box in which a Bunny is hiding. Suppose there are Bunnies in A-1 and B-1. And suppose the children ask about A-1. Box A-1 is Cold to itself, but it is Hot to B-1. The rule is: Hot overrules Cold. So Box A-1 is Hot. And the same rule applies to B-1: it is Cold to itself but Hot to A-1, so B-1 is Hot.

Unlike the game with a single Bunny, the box in which the Bunny is hiding need not be Cold. It can be Hot if it shares a side with the second Bunny.

Here are two of the children's games in progress. Each grid is either ready for a claim or within one box of a claim. Are you willing to bet your bicycle? If not, what box would you ask about? (See Games 5 and 6.)

GAME 5

	A	B	C	D
1	H	C	H	
2				
3	C		C	C
4	C			

GAME 6

	A	B	C	D
1	C	C		
2	C	H	H	
3	C			
4				

But then there came a grid that seemed to be undecidable. (See Game 7.)

GAME 7

	A	B	C	D
1				
2			H	
3	H	H	C	H
4	C	H	H	C

The children claimed that they would *not* bet their bicycles.

Alice: There could be three Bunnies, A-4, C-3 and D-4.

Alice was right. If we knew for sure that there were exactly two Bunnies — if we saw them go into the grid, for example — the answer would have to be A-4 and C-3, but the data themselves do not rule out a third Bunny in D-4. At this point Alice suggested a rule change.

Alice: Okay, let's change the rules from Hot and Cold to Hot, Warm, and Cold.

We had played a game like this with just one Bunny months earlier, and virtually no explanation had been needed. "Warm," everyone knew, meant diagonally touching.

With two bunnies, one aspect of play did need clearing up. What happens when a particular box is Hot to one Bunny and Warm to another? In Game 8, Box A-2 is Hot to the Bunny in A-1 and Warm to the Bunny in B-1. Well, because Hot overrules Warm, A-2 must be Hot.

GAME 8

	A	B	C	D
1				
2	H			
3				
4				

Similarly, if a box is Warm to one Bunny and Cold to another, it is Warm because Warm overrules Cold.

Games 9 and 10 show two of the children's games in progress. Are you willing to bet your bicycle? If not, what box would you ask about? (Each grid is either ready for a claim or within one box of a claim.)

GAME 9

	A	B	C	D
1				C
2			H	
3			H	
4		C		H

GAME 10

	A	B	C	D
1			W	
2	W		H	
3				
4		H	W	

At this point, Ivan volunteered a variation. He would write a *completed* grid on the board, and the task would be to locate the two Bunnies with certainty. After having solved similar games in which the data were released one box at a time in response to the children's questions — "Tell me about Box D-4" — the children found Ivan's task pretty simple. See how you do on these completed grids. (See Games 11 and 12.)

GAME 11

	A	B	C	D
1	C	C	H	C
2	C	C	W	H
3	C	C	W	H
4	C	C	H	C

GAME 12

	A	B	C	D
1	H	W	C	C
2	C	H	H	W
3	H	H	C	H
4	C	W	H	W

Ivan decided he would up the ante. He proposed playing the same game, but this time he would introduce one mistake. Can you find Ivan's mistake? (See Game 13.)

GAME 13

	A	B	C	D
1	C	W	H	W
2	W	H	W	H
3	H	W	H	W
4	H	H	W	C

Next Ivan proposed that the game include *three* Bunnies, and he led the class in several games in which the data were given out box by box or all at once. Game 14 shows a grid of the second type. Can you find the three Bunnies in Game 14?

GAME 14

	A	B	C	D
1	C	H	C	C
2	H	W	W	H
3	C	H	H	C
4	H	W	W	H

OBSERVATIONS

Both of us kept notes throughout the sessions. And we made a number of interesting observations:

• In general, in all the classes, all the children were engaged by the Bunny activities. The high-achieving children were enthusiastic. But many low-achieving pupils were equally enthusiastic — often even more so. Indeed, many of the most engaged students were children who had often been unsuccessful and inattentive in traditional classroom activities.

Illustration by Jem Sullivan

The children's contributions were on target and concise rather than random or diffuse.

• The children almost always worked in mixed-ability friendship groups. When someone asked for a clarification of rules, suggested another method of solution, or challenged someone else's solution (or support for a solution), the interaction was attentive and respectful. And often it was the low-achieving pupil who eagerly volunteered a solution or an explanation, almost always correctly, and always that pupil's contribution was received with attention and respect by the other children.

• In general, the children's contributions were on target and concise rather than random or diffuse. In other words, there was virtually no magical or wishful thinking. What's more, the children's contributions were offered with confidence.

• We found it striking that at this age the children were almost always willing to relinquish their deeply held explanations in response to another pupil's correct or more efficient explanation.

• Many children — including some who were almost pathologically shy or reluctant to participate — acted as the class teacher in solving problems, finding more efficient solutions, and helping other children. Laura, a shy pupil whose voice never rose beyond a whisper, often took over the class in a quiet and confident way with elegant insights and parsimonious explanations. Ivan, the boy quoted above who introduced new versions of the game, was a child with Asperger's Syndrome who had had virtually no academic success or social interaction in five years of schooling.

• Almost no review was needed when a session began after a week's interval. The children simply needed to run through the rules because the Hot-Warm-Cold game was one of several Bunny games they played.

• The enthusiasm was such that some children refused to go out for recess or to leave class when the period had finished. Some children reported bringing Bunny activities home for their parents to solve.

• Most children *owned* their solutions, and they knew that they owned their solutions — perhaps because virtually no teaching by the adults took place.

• The children tended to be economical. They asked for clarification at just the right time, and they were adept at asking the best next question and in distinguishing between the best question, a weak question, and a wasted question.

• The children invented tactics that went to the heart of the issue. For example, they discovered early on that Warm was very helpful and that if Warm occurred in one of the four corners, then one Bunny was determined with no need for further data. Such a success is of interest in light of research showing that the hallmark of good mathematical thinking is the ability to get to the heart of an issue and not be sidetracked by irrelevant details.

• As with Ivan's upping the ante when a particular level of accomplishment and difficulty had been reached, the pupils eagerly asked for harder games.

• It was striking that the children were capable of such advanced thinking, which involved logical necessity, one of the basics of mathematical thinking. This was all the more exciting because some of the children were still having difficulty with the traditional arithmetic curriculum — for example, remembering their multiplication tables and carrying out rote procedures for changing a mixed number into an improper fraction.

COMMENTS

Both of us, veterans with more than 30 years of teaching experience, felt that the children's thinking,

attitudes, and enthusiasm were extraordinary. This was not an especially well-to-do school or a school for mathematically gifted children. According to data from the U.S. Census, 21.3% of adults aged 25 or over in the town had bachelor's degrees or higher. Thirty-eight percent of children in the school qualified for free or reduced-price lunches.

What led to this apparent success? Judy Barnett commented:

> Many of the children are starved for something positive — in their school lives and in their lives in general. At sixth grade, children are expected to be "there." That is, they are expected to be functioning at grade level. There are certain topics assigned to the various grade levels — fractions here, multiplication there — and many of the children are out of phase with this lockstep march. This is like saying, "Every sixth-grader must weigh so and so many pounds or reach such and such a height." The fact is that many of the children are not "there" as expected, and the result is that many of the children have given up on schoolwork.
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> The prevailing view is that kids are empty, passive beings and that the teacher's job is to fill them with this stuff called knowledge. But watch a kid. Nothing could be further from the truth than to say that they are *naturally* empty or passive.
>
> By sixth grade, the kids know who is good in math and who is not good, and many who see themselves as not good simply shut down. We *make* them empty and passive.
>
> The fact is that many of these kids are not so good at memorizing — and much of the present school math curriculum involves memorizing — but they are very good at thinking. And when they are given the chance to think, they shine like the bright sun.
>
> That the children did well on these activities is heartwarming, and it is no surprise. I have tried to emphasize problem solving and thinking in my class and — given the results here — it seems to pay off.
>
> Why did they succeed? Because they were thinking, not spouting memorized facts. Because there was no pressure — time pressure, test/grade pressure, pressure on the teacher to cover this or that. There were no boring worksheets asking kids to do the same thing over and over. Because there were no wrong answers; that is, a child could say anything sensible, and the others would react to it and comment on it, and the child would grow stronger in his or her conclusion or alter it in response to what other kids said. Because they were thinking and the judgment of their thinking was based on whether it made sense to them and to others rather than on a teacher's assessment and red marks on a test paper.
>
> Kids at this age label themselves, and the labels are often harsh. They live up (or down) to those labels until something comes along that can change the labels.

For many of the children in this class, The Bunny Game changed those labels in important and positive ways.

Answers to the Bunny Games

Game 3. The Bunny could be in B-3 or C-2. (A-2 and B-1 are ruled out by the Cold.) Several choices, among them C-1, give information to rule out B-3 or C-2.
Game 4. The Bunny must be in A-1.
Game 5. The Bunnies must be in B-1 and D-1, assuming no two Bunnies can be in a single box.
Game 6. One Bunny must be in C-2. The second is in D-2 or C-3. Ask about C-4 or D-1.
Game 9. B-2 and D-3.
Game 10. B-3 and D-2.
Game 11. D-1 and D-4.
Game 12. A-2 and C-3.
Game 13. Ivan changed A-4.
Game 14. A-1, A-3, and D-3.

1. Background on the principles of such educational activities as The Bunny Game is given in two prior articles. See Thomas C. O'Brien, "Some Thoughts About Treasure Keeping," *Phi Delta Kappan*, January 1989, pp. 360-64; and idem, "Parrot Math," *Phi Delta Kappan*, February 1999, pp. 434-38. Several Bunny Games are available as software produced by Thomas C. O'Brien for Palm OS devices under the title "Treasure Hunt." They can be found at the Handmark and Handango websites. K