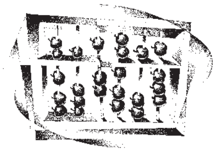
REAL MATH?



Children shouldn't memorize mathematical facts and procedures like parrots, but should learn to think mathematically. Mr. O'Brien and Ms. Moss stress that mathematics involves making sense. They provide some simple mathematical challenges for readers to make sense of and to try with their classes.

BY THOMAS C. O'BRIEN AND ANN MOSS

THE FIRST author had been working with the staff of an elementary school, showing them a logical-inference activity involving "Bunnies" that some *Kappan* readers might remember.[1] It was an activity that he had used in a guest session with fifth-graders in the school earlier in the day.

"When are you going to do some *real* mathematics?" a teacher asked.

"Say more?" he countered.

"You know, *real* math, like addition and subtraction and multiplication and division. Dead Mice Smell Bad. You know, DMSB: Divide, Multiply, Subtract, Bring down. That's how you do division."

Of course, we can all agree — as we would all vote for Mom's apple pie — that children should certainly engage in *real* math in schools. But the question must be asked, "What is real mathematics?" Certainly it's not the rote memorization of arithmetic "facts." Parrots can say things like "six times three is 18" without knowing a thing about six or three or times or 18. Are they doing real math?

The fact is that an exclusive emphasis on rote memory and rote performance of computational procedures at a time when every desktop computer can do billions of computations in a second is downright foolish. Of course, children should learn to add, subtract, multiply, and divide, and they should do so sensibly and efficiently. And while shopkeeper's arithmetic is important, it is a very small part of real math.

> Children today are facing a world that is shaped by increasingly complex, dynamic, and powerful systems of information in a knowledge-based economy. . . . Being able to interpret and work with complex systems involves important mathematical processes that are underemphasized in numerous mathe-

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> matics curricula, such as constructing, explaining, justifying, predicting, conjecturing, and representing, together with quantifying, coordinating, and organising data.[2]

Above all, school mathematics should involve *making sense*.

Rather than just read on and on and perhaps merely store some text as parrots do, we invite readers to *engage* the issue. (In fact, the teacher who asked about *real* math was asked to work on some Bunny problems and then to try them with her pupils.)

Below, we offer some activities for teachers to try — preferably in a group problem-solving situation — and then use with their students. The activities involve issues that are profoundly important and profoundly mathematical. They challenge the problem solver to make sense.

Rather than deal with a wide array of mathematical activities, we will limit the activities presented here to two very fundamental ideas: classification and inference. By classification, we mean "putting things into mental baskets." The ability to classify is utterly fundamental in human thinking. If you doubt this, just spend some time with an Alzheimer's patient.

By inference, we mean "generating new knowledge out of old knowledge." Hold a coin behind your back, put it in one hand, close your fists, and show them to a child. Ask, "Which hand?" The child who points and, finding that one hand is empty, says with certainty that the other hand holds the coin is drawing an inference. That child has generated new knowledge (this hand holds the coin) from old knowledge (that hand is empty). Moreover, the new knowledge is *necessary*. On to the examples.

CLASSIFICATION: PRESCHOOL AND EARLY ELEMENTARY SCHOOL

Here are some "games" (a word not frequently heard in education circles these days) and simple follow-up activities to choose from.

• Say to the child, "Here is something green." (The teacher or caregiver holds up or points to something green.) Then ask, "Can you show me something else that is green?" Now here are some options to follow up with.

1. When the child has found something green, ask, "Can you tell me something else that's green?" Keep going for two or three steps.
2. Use other colors — red, blue, white, etc.
3. Play the same game with a group of children, but this time have them answer in words with their eyes closed.

• Say to the child, "I am thinking of an animal." (The only animals allowed in this game are cats, dogs, cows, and lions.) Each child, in turn, makes an animal noise — meow, woof, moo, or roar. If the noise is the one that the secret animal makes, that player is the winner and another round is started, with the winner choosing the next animal. If the noise is not the noise of the secret animal, the next child takes a turn guessing. Some options:

1. Change the animals to a different set of four.
2. Go as high as five or six animals and allow the children to ask questions instead of making a noise. For example, Does it fly? Is it found in a zoo?
3. Allow members of the class or of the child's family (if they are known to all the players) to be "secret" things.

What is all of this about? Putting things in mental baskets, that is, classifying. Imagine what life would be like if you had to deal with everything on an item-by-item basis. Life would be chaos. A second important aspect of these games makes available to children the essential nature of inference: the canceling of items on a list (the wrongly guessed animals) until you get down to one.

CLASSIFICATION AND LOGICAL NECESSITY: ELEMENTARY AND MIDDLE SCHOOL

• Play the "I am thinking of a person" game, where one player thinks of a person and the others ask questions in turn. You can try all sorts of options.

1. The person must be present in the classroom.
2. The person is someone local and known to all, but not present in the classroom.
3. The person is connected with the school curriculum — for example, a historical figure.
4. The person comes from popular culture — for example, someone in the entertainment world.
5. Play "I am thinking of a state" (or a nation), where players have access to a map, and the only questions allowed are in the form of "Does it touch Alabama?" or "Does it touch Italy?"
6. After playing several "I am thinking of . . ." games, list some categories on the chalkboard, and challenge the players to the game "I am thinking of a category." Here a category (such as "alive" or "American" or "female" or "author") is to be deduced, and the players ask such questions as, "Does Elvis Presley fit the category?" The task is to infer the category.

Whatever activity you choose, make sure that, as far as possible, children get the chance to be the information-givers as well as the question-askers.

What's it all about? Activities like these involve classification, and they give children a chance to practice canceling things until they get down to a final item. This ability lies at the heart of logical thinking, both in mathematics and in everyday life.

The fancy name for it is logical necessity, and it is concerned with the distinction between *must be* and *might be*. In the games above, the task is for the child to eliminate the various items on a list and get down to one. One definition of logic that stands up pretty well despite the *p*'s and *q*'s you may have met in university logic courses is "the systematic generation of alternatives, perhaps to exhaustion (i.e., get 'em all) and the systematic canceling of alternatives, perhaps to uniqueness (i.e., cancel 'em down to one)."

These two processes are utterly basic in thinking. Indeed, they were what Jean Piaget, the now widely ignored epistemologist, studied in children for over 60 years.

PATTERNS AND LOGICAL NECESSITY: ELEMENTARY AND MIDDLE SCHOOL

Here are some more games involving patterns and logical necessity. We present the activities in order of age level, but our experience has shown that the first few activities, while accessible to young children, remain engaging for many pupils at the upper-elementary level.

• Ask children to line up in a pattern such as boy/girl/boy/girl.

1. Can you think of another possible pattern? (For example, every third person is a boy.)

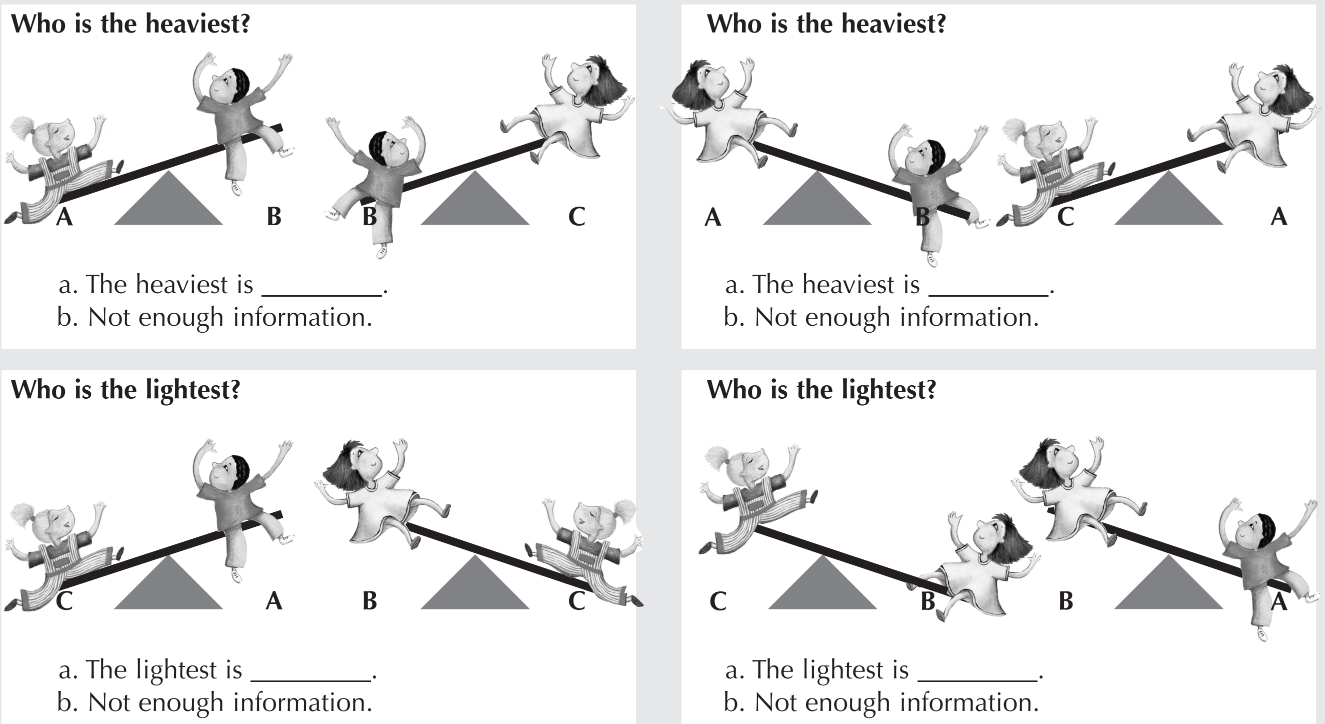
2. Imagine a pattern where children line up boy/girl/boy/girl. Then imagine that the children shuffle around to make a boy/boy/girl/boy/boy/girl pattern. Can some children keep their original places?

• I say, "One," and you say, "Two," and we keep taking turns with consecutive numbers. Who gets to say 15? How about 112? How about a million?

Recent work with children in grades 4 and 5 suggests that this game is very easy. You are simply putting principles (odd and even) to work so that you don't have to do all the work yourself. But try it with three or four people saying consecutive numbers. Now, "Who says 153?" becomes a more challenging question. For many children this is difficult, and for parrot-trained children, it is close to impossible.

• Two people take turns counting out loud: 4, 8, 12, 16, and so forth. Can you tell, without counting that high, who will get to say 21? (Nobody says 21. Give 21 to the dog.) How about 32? 100? A million?

FIGURE 1.
An Inference Problem



 Illustrations: EyeWire Images

1. Try skips of 3 or 5 or 6, rather than 4.

2. Try the same thing with four or five people counting by turns.

What is this all about? Recent work shows that the count-by-fours task (with two people) is not so easy for many children at grade 4. They tend to make long lists. Is there a principle you can put to work so that you don't have to do all the work yourself? Finding such a principle can be a rewarding process for the children. What they are seeking is to find some underlying sense, rather than to rely on a laundry list of information.

• Three children — A, B, and C — sit on a seesaw. (See Figure 1.) Who is the heaviest, A, B, or C? Explain your thinking.

You can make up some similar problems, of course. Can the children? Tryouts showed that fifth- and sixth-graders do wonderfully well with these problems and are very successful at making up new and original ones.[3]

MORE LOGICAL NECESSITY: MIDDLE SCHOOL

The following activity is far more open-ended than the other ones we've described to this point. Readers might try to see how far they can go with it.

• Here's a calendar for a certain month. (See Figure 2.) What number patterns can you find? If you looked at a different month, would these same patterns occur?

Let's revisit the count-by-fours question. This version is a bit more advanced.

• Al, Bob, Carla, and Diane say the numbers in order. The numbers jump by 4s.

A	B	C	D
4	8	12	16
20	24	28	32
36	40	44	48

If the pattern continues, in which column (if any) will the following numbers appear: 88, 100, 122, 137?

1. If you think that this one is too simple, try starting the grid off out of phase and wrapping in different directions when the rows change.

A	B	C	D
1	5	9	13
29	25	21	17
33	37	41	45

Now, where, if at all, would you find 88, 97, 103, 211?

2. Ask the students themselves to make up problems like these and try them out on one another.

• Back to calendars once again — and logical necessity. (Problems such as these will challenge anyone at any age.) Ms. Jones has a calendar for all the months in the period 2000-2099. Can she find a month that has:

a. Five Sundays?

b. Exactly three Tuesdays?

We mentioned at the outset that mathematics in school should be about *making sense*. That's true. But in an educational setting, where one person's task is to help another person to grow, there is another important goal as well. School mathematics is not only about making sense (as children can do when listening to a story read to them), but also about facing the *challenge* to make sense.

By challenging children, we cause them to construct or extend or revise their mental networks. In life in general, this mental growth occurs when we encounter and solve problems. Thus in the classroom, we should constantly pose problems for children to solve. Problem solving in mathematics means much more than finding answers to routine computational exercises, even when they are couched in words. The great mathematician George Polya once said to the first author, "A problem is a difficulty which you can't immediately resolve. Suppose you are hungry late at night, and you go to the refrigerator, and it is empty. *Then* you have a problem."

The essential Piagetian notion is that learning is provoked adaptation. Leslie Steffe and Ron Tzur say it simply: "[Learning is] the capability of an individual to change his or her conceptual structures in response to perturbation."[4]

We hope that the activities we have suggested will give teachers (and children) some sense of what real mathematics is. For those readers who would like to have access to more mathematical problems for schoolchildren, we've added an annotated list titled "Additional Sources for Problems" (page 296). And we hope that solving the problems here and the ones that can be found in the resources we recommend will give teachers a glimpse of learning as

FIGURE 2.
What Patterns Do You See?

S	M	T	W	TH	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

If you looked at a different month, would these patterns occur?

provoked adaptation. George Polya said it succinctly 50 years ago:

> A teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations, he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge and helps them to solve their problems with stimulating questions, he may give them a taste for, and some independent means of, independent thinking.[5]

(Continued on page 303)

Additional Sources for Problems

Tim Erickson, *Get It Together: Math Problems for Groups, Grades 4-12*. Publisher: EQUALS, University of California, Berkeley, Lawrence Hall of Science, Berkeley, CA 94720-5200. Date: 1999. ISBN 0-9125-1153-2. Available from amazon.com.

Comments. *Get It Together* is a collection of mathematics problems for group problem-solving. Since each member of the group has a different bit of information, everyone has to work together to arrive at a solution. Some of the mathematics topics covered are logic, numbers and operations, geometry, algebra, probability, measurement, and functions.

Tim Erickson, *United We Solve: 116 Math Problems for Groups, Grades 5-10*. Publisher: eeps Media. Date: 2002. ISBN 0-9648-4960-7. Available from amazon.com.

Comments. A collection of math problems for groups to solve together, thus promoting thinking and interpersonal collaboration. Additional materials are available online.

Claudia Zaslavsky, *Math Games and Activities from Around the World*. Publisher: Chicago Review Press. Date: 1998. ISBN 1-5565-2287-8. Available from amazon.com.

Comments. More than 70 math activities from all parts of the world, with historical and cultural background information, encourage children to develop their thinking in various branches of mathematics. Math, history, art, and world cultures work together in this classic book. Other Claudia Zaslavsky books include *More Math Games and Activities from Around the World* (ISBN 1-5565-2501-X), *Number Sense and Nonsense: Building Math Creativity and Confidence Through Number Play* (ISBN 1-5565-2378-5), and *Africa Counts (Number and Pattern in African Cultures)* (ISBN 1-5565-2350-5).

Akihiro Nozaki, *Anno's Hat Tricks*. Publisher: Harcourt Brace. Date: 1993. ISBN 0-1530-0349-9. Available from amazon.com.

Comments. One of Anno's many classics! Three children, Tom, Hannah, and Shadowchild (who represents the reader), solve logical problems concerning the color of the hats on their heads. An introduction to logical thinking and mathematical problem solving. See also *Anno's Mysterious Multiplying Jar*, *Anno's Math Games*, *Anno's Math Games II*, and *Anno's Counting Book*, available from various third-party sources.

Dale Seymour and Ed Beardslee, *Critical Thinking Activities in Patterns, Imagery, Logic*. Publisher: Pearson Learning Group. Date: 1997. ISBN 0-8665-1440-6. Available from amazon.com.

Comments. Helps children recognize patterns, use visual imagery, and reason logically. Children explore many topics while finding winning strategies, applying deductive logic, and classifying. Other Dale Seymour books available from Pearson include *Favorite Problems* (ISBN 0-8665-1085-0), *Estimeasure: Estimation & Measurement Activities* (ISBN 0-7690-1024-5), *Problem Parade* (ISBN 0-8665-1207-1), and *Visual Thinking* (ISBN 0-2016-8648-1).

Tom O'Brien, *Daily Tantalizers® Math*. Publisher: ETA/Cuisenaire. Date: 2000/2003. ISBN 0-7406-0100-8.

Comments. Each of eight books (roughly grades 1-8 but used in high schools as well) provides 180 problem-of-the-day activities, ranging from arithmetic to visual thinking to algebra to logic. Many of the activities are novel, and many have multiple methods of solution. Other Tom O'Brien books available from ETA/Cuisenaire include *VersaTiles® for Middle Grades Critical-Thinking* (set of five books) (ISBN 0-7406-0828-2; 0-7406-0830-4; 0-7406-0832-0; 0-7406-0834-7; 0-7406-0836-3).

Jean Kerr Stenmark, Virginia Thompson, Ruth Cossey, and Marilyn Hill (illustrator), *Family Math* (available in English, Spanish, Chinese, and Swedish). Publisher: EQUALS Programs, Lawrence Hall of Science, University of California, Berkeley, CA 94720-5200. Date: 1986. ISBN 0-9125-1106-0. —TO'B and AM

Real Math?

(Continued from page 296)

1. Thomas C. O'Brien and Judy A. Barnett, "Fasten Your Seat Belts," *Phi Delta Kappan*, November 2003, pp. 201-6; and idem, "Hold Onto Your Hats," *Mathematics Teaching*, May 2004, pp. 9-12.

2. Lyn D. English and James J. Watters, "Mathematical Modeling with Young Children," *Proceedings of Psychology of Mathematics Education*, vol. 28, 2004, in press.

3. Thomas C. O'Brien et al., "Teachers as Researchers: The Bear Problem," *Philosophy of Mathematics Education Journal*, November 1999. Available at www.ex.ac.uk/~PErnest.

4. Leslie P. Steffe and Ron Tzur, "Interaction and Children's Mathematics," in Paul Ernest, ed., *Constructing Mathematical Knowledge: Epistemology and Mathematical Education* (London: Routledge/Falmer, 1994.)

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