HOLD ON TO YOUR HAT

Thomas O'Brien and Judy Barnett

In MT185 Tom and Judy invited readers to 'Fasten their seat belts' whilst trying some problem-solving with KS2 pupils. In 'Hold on to your hat' Tom and Judy develop this idea, again inviting readers to engage in the activities rather than merely reading them.

Introduction

For several months a veteran teacher and a university professor have been conducting occasional problem solving sessions with three of the teacher's sixth grade (age 11-12) mathematics classes in the teacher's elementary school. The results have been surprising and the activities have given rise to several articles over the past year. [See Footnote 1 and References.]

The latest problem session took place after a dozen sessions or so involving similar activities. The issue was inference, the notion that one can *derive* new information from old information with logical certainty. Put a coin in one hand, make fists, show the fists to a child at age five or so, and ask the child to find the coin. The child will find the coin with certainty. She may see the coin with her eyes or if she is shown an open hand she can see the coin with her *mind*.

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The Bunny Game Version 2

In our work with children, we called all the various games 'Bunny Games' – a Bunny is hiding in a 4-by-4 grid and your job is to find him. In the activities reported here, the Bunny Game involved distance. In the first game, one Bunny was hidden at random in the grid. [See *Figure 1*.]

We said to the children: *Your job is to find the Bunny. Ask about one of the boxes. For each choice you make, you will be given the shortest distance from the box you choose to the box where the Bunny is hiding.*

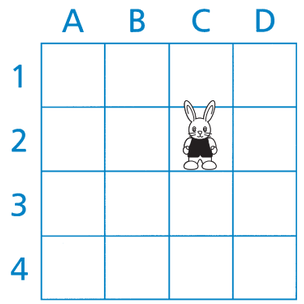


Figure 1

You will not be given diagonal distances, only left-right and up-down.

When you think you have figured where the Bunny is, tell us that you need no further information. At that point tell the class the location of the Bunny. If you are wrong, you are out of the game for that round.

Lisa: "What happens if we ask about the box where the Bunny is hiding?"

Teacher: "The distance is zero."

Here is a sample of the three or four games that were played. Three questions were asked and the answers were marked in a grid on the chalkboard. [See *Figure 2*.]

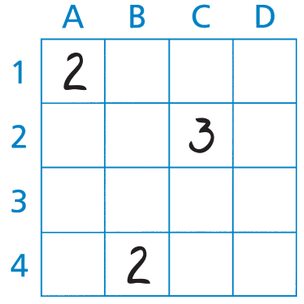


Figure 2

Can you tell where the Bunny is? Yes, A-3. After a half hour's experience with grids of this type, children were able to solve this one instantly. Do you need all the information that's given? Children were quick to say that the data were redundant; 'the 2 in B-4 is not needed'.

Here are some similar games. Can you find the Bunny? Redundant information? Not enough information? [See *Figure 3*.]

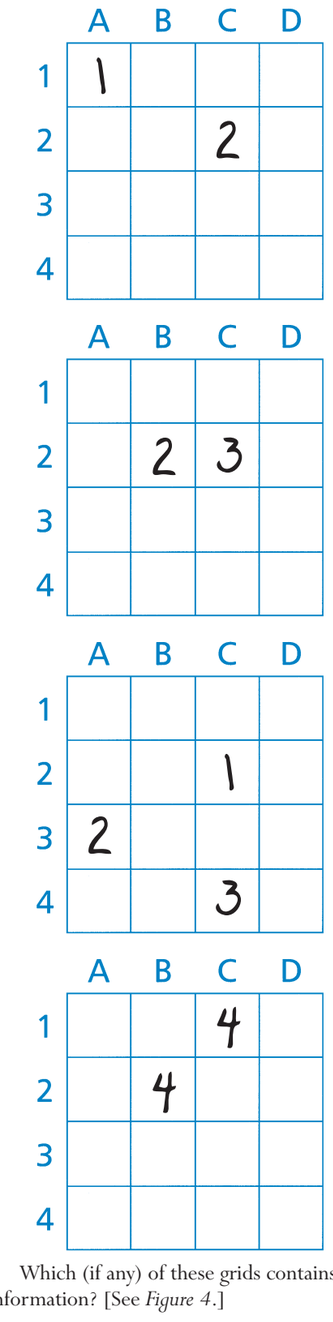


Figure 3

Which (if any) of these grids contains faulty information? [See *Figure 4*.]

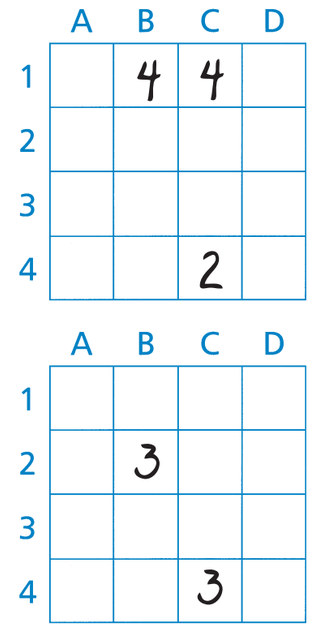


Figure 4

Children were successful at this and were anxious to go on to two Bunnies.

The Two-Bunny game

We said to the children: *In the Two-Bunny game, you are first given the distance from the box you choose to a randomly chosen Bunny. (However, if the first distance is zero, you are given the distance to the other Bunny.) If you choose the same box again, you are given the distance to the other Bunny.*

When you think you have figured where both Bunnies are, tell us that you need no further information. At that point tell the location of the Bunnies. If you are wrong, you are out of the game for that round.

The first question asked was B-2. The response was 4. Children instantly recognized that one Bunny was in D-4. [See *Figure 5*.]

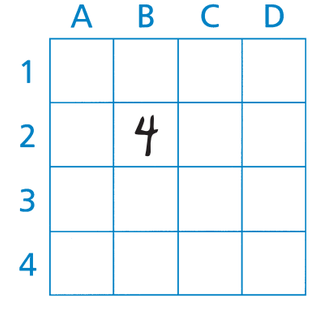


Figure 5

The second question was B-2 again. The second Bunny got a 1. [See *Figure 6*.]

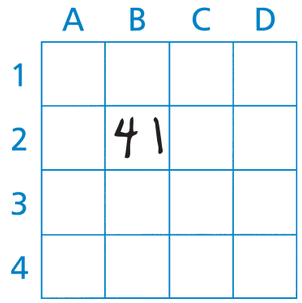


Figure 6

At this point one of the pupils suggested drawing an auxiliary grid. 'B is for the Bunny we are sure of.' [See *Figure 7*.]

"What's the C?" we asked.

"C is for '**C***ould be*'."

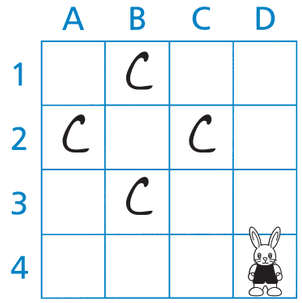


Figure 7

What's the best question now? C-2 and D-4 were the two choices and after some discussion, C-2 won. [See *Figure 8*.]

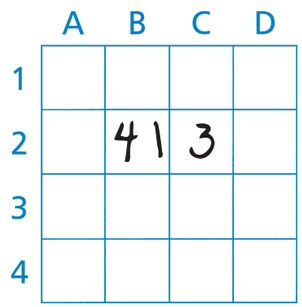


Figure 8

'That's not helpful,' children said, 'because the 3 must refer to the Bunny we've already found.'

'Must or may?' we asked. 'Must,' they said with certainty.

The next choice was C-2 again. This time they got a 2. [See *Figure 9*.]

'Still not finished,' was the universal response. 'How about C-3?' C-3 drew a 1 and the children

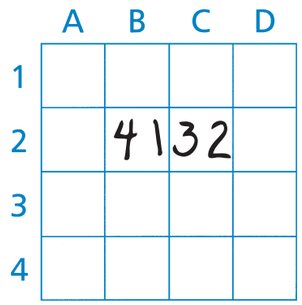


Figure 9

instantly saw that the game was over.

The entire session took no more that fifteen minutes. Children were well on their way to owning the Two-Bunny game.

Here are four grids which occurred (one question at a time) in the next session. In each case can you tell how many Bunnies are determined? [See *Figure 10*.]

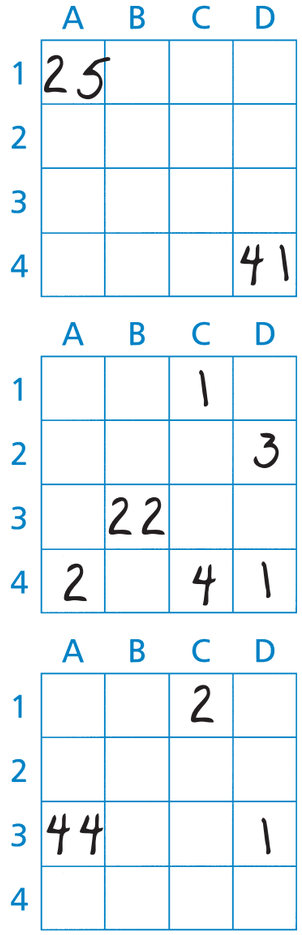
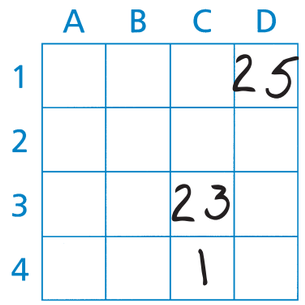


Figure 10



And if more information is needed, what next question would you ask?

Observations

Both teachers kept notes throughout the sessions, not only of the children's moves during the games but also of the classroom dynamics. What we observed should not surprise veteran child watchers. But to those who usually focus on mere questions and answers and not on the child, we say, 'Hold on to your hat.'

- As was noted in the observations in **MT185**, Children with high achievement levels performed well but the 'low achievers' were equally successful in solving the games despite their often being unsuccessful and inattentive in 'regular' classes. Much of our traditional mathematics curriculum and our tests involve applying routine procedures – often computational procedures – rather than inventing procedures and making sense of the procedures and the results they give. Here, children were at all times encouraged to come up with their own procedures. If their own procedures didn't work they changed their procedures in the face of feedback from other children. Evaluative or instructional feedback from the teachers was virtually nil.
- Different groups often came up with different procedures. Then in discussion and explanation, groups would be swayed or convinced by what other groups had to say. For example, the use of double digit feedback with two Bunnies was a critical invention of one of the groups and other groups adapted their thinking to incorporate this new and important tactic.
- Likewise (**MT185**) Children worked in mixed ability friendship groups. They co-operated, not competed, with one another. They asked one another to clarify a choice of box. They suggested alternative choices and they respected the suggestions of the 'low achievers' who historically had had little success in mathematics. In fact, they recognized that those suggestions were often more insightful that those offered by the 'smarter' children.
- Contrary to what often happened in the traditional mathematics classroom, children didn't call out suggestions randomly. Instead, they analyzed the grids, discussed the next best move based on the evidence they had collected, and then made well-informed moves. Very few moves were weak or wasted moves.
- They were almost always willing to relinquish their own suggestions when another child proposed a move they could see as more efficient.
- Similarly (**MT185**) the child with Asperger's Syndrome, was often the first to solve the Bunny problems and his solutions were often more efficient than those proposed by others. The children quickly looked to him for clarifications, explanations and next steps. He often went to the chalkboard spontaneously to explain his reasoning to the entire class and several times he invented classroom activities and carried on discussions as though he was the teacher.
- Another shy pupil, whose voice never rose beyond a whisper, routinely spoke up forcefully with elegant insights and parsimonious explanations – something quite unusual in all her school history.
- As in the Bunny games reported earlier, children invented tactics which went to the heart of the issue. For example, children saw that it was very helpful to have two distances for a single box rather than two single numbers in different boxes, (two numbers in a single box had to be independent of one another). One group would ask for a box and the next group would ask for the same box.
- The pupils eagerly asked for more difficulty in the apparent belief that they could conquer any level of complexity, even a three or four Bunny game.
- As before, children tended to be economical. One way of solving the distance activities is to ask about every box in order to see where the zeroes show up. No one did this. No one searched for zeroes at all because they knew that they could find both Bunnies with as few as four or five bits of data. They continued to be so engaged with solving the problems that many refused to go out for recess or to leave class when the time period had finished and many children played the Bunny activities at home with their parents and brothers and sisters.

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Comments

Both teachers were frequently amazed by the complexity of children's thinking. Often the children were far ahead of the adults in speed and complexity of thinking. The children often reached a conclusion ahead of the teachers and the adults had to slow down the pace to analyze what the children were saying.

More general, the activities of the children seemed to give support to the Piagetian notion that a major factor in learning is provoked adaptation. Pose a challenge which is moderately beyond their reach and children – people in general – will commonly revise, adapt, extend, and elaborate their existing networks of ideas and actions and construct new ones in order to accommodate the mismatch between the task demands and where they stand at present. They do so by evolving in terms of coherence, economy, stability, generalizability accompanied by a tendency to quest. (In this case, 'quest' involved pleas for more challenging games.)[1]

Barnett commented, 'Let me be clear. These children were not special kids in their schoolwork or their behaviour. Some of the boys would think nothing of pushing another boy's face into the toilet if they didn't like his looks.'

In reflecting on the observations we made, Barnett continued, 'Do you realize the freedom we gave kids? How this empowered them! It's like asking someone for his opinion. That empowers him. Such an attitude creates security and it opens up doors to all sorts of positive feelings children have for themselves (and teachers have for children) and for their own learning.'

Work on these activities continues, their presentation made somewhat easier by the fact that they are now available for Palm pdas (www.handmark.com). We are looking to see how early in age the present mastery might occur and we are gathering data on the sticking power (after six months) of the original groups' learning. Readers are invited to try these activities with children and correspond with Tom at 74055.652@compuserve.com or Judy at maxbudda@yahoo.com.

Tom O'Brien is a consultant and author. He lives in Chesterfield, Missouri. Judy Barnett recently retired as a teacher at Renfro Elementary School, Collinsville, Illinois, USA.

[1] See ***Mathematics Teaching 185***, December 2003, for Tom and Judy's first article, 'Fasten your seat belts,' and a description of the Hot-Warm-Cold Bunny game.

References

1 Thomas C. O'Brien: 'Some thoughts about treasure keeping;' ***The Kappan***, January 1989

2 Thomas C. O'Brien & Judy A. Barnett: 'Fasten your seat belts,' ***Phi Delta Kappan***, November 2003

3 Thomas C. O'Brien & Judy A. Barnett: 'Fasten your seat belts'; ***Mathematics Teaching 185***, December 2003