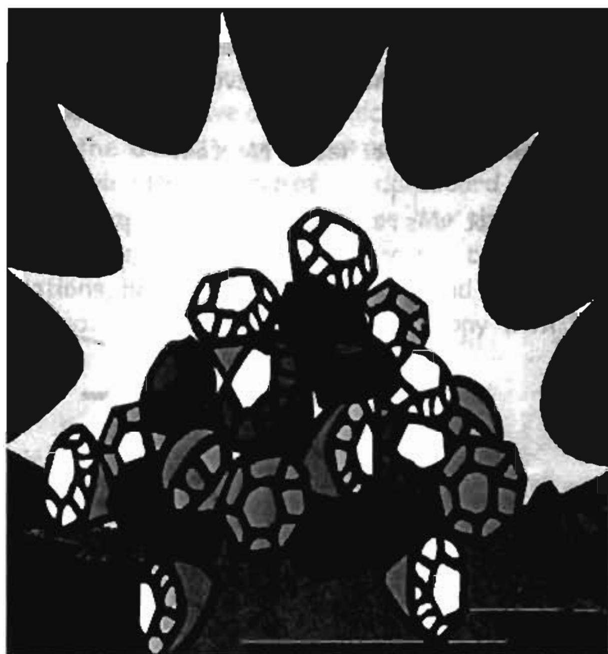
Children's Construction of Necessary Inference



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For several years we have been investigating children's performance on inference activities invented by O'Brien and coded for Palm handheld computers. By inférence, we mean the generating of new knowledge based on old knowledge. Moreover, we are concerned with necessary inference, new information which, given the data, *must* be true.

What is necessary inference? Necessary inference is beginning to be constructed by pre-school children. Put a dime at random in one of your hands, close both hands, and show both fists to a child asking, "Which hand?" If the child chooses the hand with the dime and the fist is opened, the game is finished. If the child chooses the empty hand and the fist is opened, the game is also finished. The child cannot see the dime but her *mind* can infer the dime. This is a primitive game involving necessary inference. Inference, including necessary inference and plausible inference (where the conclusion is likely but not certain), are utterly basic to everyday thinking and of mathematical thinking. They do not get much play in most American school mathematics curricula. Although the activities described here are available via a handheld computer, they do not require a computer. They can be conducted by a teacher using a chalkboard with any number of pupils. In fact, after a bit of warm-up, children can conduct the game among themselves.

In particular, this research is concerned with systematic cancelling, a fundamental aspect of logical thinking. Suppose, for example, that you were playing a game called, "Mystery Number from 1 to 10," and suppose a pupil asked, "Is the Mystery Number less than 5?" If you answered, "No," children would cancel certain numbers—mentally or by crossing them out (as shown below)—as a consequence of the answer you gave.

~~1 2 3 4~~ 5 6 7 8 9 10

Our latest observations took place with a game called Rubies in a suite of games called Treasure Hunt.

The suite consists of three games in all, Rubies, Emeralds and Diamonds and each has two levels, one jewel or two jewels. In all the games a jewel is hidden at random on a 4 by 4 grid and children have to generate questions, gather clues, and weave them together to make inferences in order to tell with certainty where the jewel is. The investigation took place in two fifth grade classes (children's age: 10) in elementary schools in a major metropolitan area—one public and one private—in which thinking and investigation and the construction-of-ideas were encouraged and learning-by-transmission-of-information-by-teacher very rare.

Children worked in teams of two or three of their own choosing. They were asked to think about various questions to ask the computer and, after the teacher provided information from the computer, they were called upon by the teacher to generate and discuss the consequences of the feedback, along with justifications and comments. That is, students asked for a particular square in the 4 by 4 grid and the teacher put that location into the computer and reported the computer's response back to the class. Children then had to figure out the consequences of the information provided to them. As with all our prior research, no teaching whatsoever took place except that the rules of the game were announced and children were encouraged to keep a Consequences Grid—i.e., a grid showing which boxes had been ruled out by the data or which were possible (or certain) boxes for the jewel (or both), as decided by the children. Further, no comment was made by the teachers as to whether or not the consequences derived by children were correct. The activity was entirely in children's hands except for the reporting of data from the computer.

In Rubies, children ask about a box and the computer looks in that box and all the boxes that touch that box *on a side or at a vertex* to see if it sees a Ruby. For example, if children cite B-2, the computer looks in all the shaded boxes. If it sees a Ruby it reports 1, otherwise 0.

	A	B	C	D
1	■	■	■	
2	■	■	■	
3	■	■	■	
4				

The game of One Ruby

		0	

Here is an example of children's work:
Teacher: What box would you like to ask about?
Pupil: C-3
T: The computer looks in C-3 and all the boxes which touch C-3. It sees 0 Rubies. (The teacher or one of the pupils listed the feedback on a data grid like the one below.)
Pupil. That means a lot of boxes are zapped.
T: How many?
Pupil. Eight.
Pupil: Nine.
T. Draw a Consequences Grid and mark the boxes which have been zapped.

Pupil. Draws this

Consequences

	X	X	X
	X		X
	X	X	X

Pupil. But C-3 is also zapped. The computer looked in C-3 and all the boxes which touch C-3 and reported 0. There can't be a Ruby in C-3.
Pupil. OK. Adds an X to the Consequences Grid

Consequences

	X	X	X
	X	X	X
	X	X	X

T. What's a good box to ask about now?
Pupil. A-3
T. A-3 is a 1.
T. Can you show the consequences?
Pupil. The new information zapped four more boxes. Updates the grids as shown.

	A	B	C	D
1				
2				
3	1		0	
4				

Consequences

X	X	X	X
	X	X	X
	X	X	X
	X	X	X

Pupil. So there are two ways that a box can get zapped?
T: Say more?
Pupil: In the beginning boxes got zapped by a 0. They were in or touching a box with a 0. But all of row A just got zapped even though none of its boxes is touching a 0.
Pupil So?
Pupil. The 1 we just got for A-3 means that the Ruby must be in A-3 or one of the boxes which touch A-3.
Pupil. Right. So unzap box B-3.
Pupil. No. Once a box is zapped it stays zapped. The Ruby can't be in B-3 because of the 0 in C-3.
Pupil. OK, I understand.
Pupil. Because of the 1 in A-3, the Ruby must be in A-2 or A-3 or A-4. That allowed us to zap all of row 1.
Most of the class says, "Yaaaaay."
T. What next?

Pupil. What about A-2?
T. A-2 is a 1.
Pupil updates the grids as shown.

	A	B	C	D
1				
2	1			
3	1		0	
4				

X	X	X	X
	X	X	X
	X	X	X
X	X	X	X

Pupil. Hold it. Don't zap A-4. The Ruby could be in A-4.
Pupil. Why?
Pupil. Because A-3 is a 1. And A-4 touches A-3.
Pupil. But the 1 in A-2 refers to A-2 and all the boxes which touch A-2. And A-4 doesn't touch A-2.
Pupil. OK. I see what you mean.
Pupil. So now all we need to ask A-1. If A-1 is a 1, the Ruby is in A-2. If it is a 0, the Ruby is in A-3.
Several pupils: There are other boxes you could ask which are just as helpful as A-1.
Pupils named B-1 and B-4 and A-4.
Quite correctly, nobody named A-2 or A-3 or B-2 or B-3, none of which would have given new information.
What we have shown is a case of construction of an inference tactic by fifth graders. The notion that there are two different ways to zap a box was noteworthy, we thought. If a box gets a 0 or touches a box with a 0 the Ruby cannot be in that box. But there is a second way to zap a box. If a 1 has occurred any box that does not contain that 1 or is not touching that 1 box can be zapped.

Two Rubies

The notion that there are two different ways to zap a box arose with the class in the second school, this time in a Two-Ruby game. This group, also fifth graders and also working in teams of two or three, was playing its first Two-Ruby game after having had an hour's session with One Ruby a week earlier.
The first two choices were A-1 and D-4, both of which were 0.
Then A-4 turned out to be 1.

	A	B	C	D
1	0			
2				
3				
	1			0

Consequences

X	X		
X	X		
		X	X
		X	X

Pupil. That means that one Ruby is in the top right 2 by 2 and the other in the bottom left 2 by 2.
T: Why?
Pupil. A-4 would have to have been a 2 for both Rubies to be in the bottom left corner's 2 by 2. I would ask about B-2 or C-3. What I'm hoping for is a 0 or a 2.
Teacher: C-3 is a 1.
Pupil. Aaaargh!
Pupil. Let's try B-1. That would help us know about two boxes.
B-1 was a 0 and the pupils zapped C-1 and C-2.

	A	B	C	D
1	0	0		
2				
3			1	
4	1			0

Consequences

X	X	X	
X	X	X	
		X	X
		X	X

What's going on here? Not much, except for a ghastly set of data. But the fireworks are just ready to begin.

Pupil: Tell us about A-2?
T: A-2 is a 0.
Pupil: And C-3? That was a 1, right?
T: C-3 was a 1.
Pupil. We're finished. The 0 in A-2 zapped A-3 and B- 3. And the 1 in C-3 tells us that a Ruby must be in B-4.
Pupil: The 1 in C-3 must refer to the Ruby we just found, B-4. This means that there is no Ruby in D-2, or else C-3 would be 2. The second Ruby must be in D-1. (See Note 1.)

	A	B	C	D
1	0	0		
2	0			
3			1	
4				0

Consequences

X	X	X	R
X	X	X	
X	X	X	X
	R	X	X

By now pupils had spent about 25 minutes on this Two-Ruby game.
"Let's try another one," they said eagerly.

This time the draw of data was much more quickly revealing.

C-1 drew a 2 and B-3 drew a 0. The 2 zapped ten boxes (all of rows 3 and 4 and A-1 and A-2) and the 0 zapped another two (B-2 and C-2).

	A	B	C	D
1			2	
2				
3		0		
4				

Consequences

X			
X	X	X	
X	X	X	X
X	X	X	X

Pupil: How about B-1?
T: B1 is a 1.
Pupil: And D-1?
T: Also a 1. Do you think we're finished?

	A	B	C	D
1		1	2	1
2				
3		0		
4				

Pupil: No. I'd like to know about D-2
T: D-2 is a 1.
Pupil. OK. We're finished,
T: Say more?
Pupil. The 1 in B-1 means that a Ruby has to be in B-1 or C-1. Can't be both or else because B-1 would have to be a 2.
T: OK.
Pupil. And the other Ruby has to be in D-1 or D-2.
T. OK.
Pupil. So the first Ruby—the one that must be in B-1 or C-1—has got to be in B-1 because if it were in C-1 then D-1 would have to be a 2.
T: OK. Go on. How do you get the second Ruby?
Pupil. If we know that the second Ruby is in D-1 or D-2 all we need to ask is D3. If D-3 is a 1, the second Ruby is in D-2. If it's a 0, the second Ruby is in D-1.
T: D-3 is a 1.
Pupil. OK, we're finished. The Rubies are in D-2 and B-1.

	A	B	C	D
1		1	2	1
2				1
3		0		
4				

Conclusion

What you see here is children's construction of thinking involving fundamental aspects of logical necessity. As with our prior research, we found children's thinking to be very complex and economical, and children never asked a useless question and very rarely made a false inference. Moreover, as with prior research, we found that children worked together with enthusiasm and respect and co-operation and we found that virtually all children in both classes were successful, not just the high achievers.

The notion that knowledge is constructed is not popular in these days. Indeed, a denial of this fundamental human act—perhaps the must fundamental cognitive act of all—has led educational critics of mathematics education, many of them with no experience in mathematics or [inclusive or] with children, to give us a fundamentalist absolutist approach to education appropriate only for parrots. (See Reference 3.) But here you see children constructing mathematics, false starts and giant leaps and all. The notion that the construction of ideas is opposed to the finding of correct answers is preposterous. Indeed, the construction of ideas is basic to achieving correct answers and applying them and generalizing them unless, of course, one views parrot-like imitation as an end in itself. The report here adds evidence to the idea that knowledge evolves in terms of coherence, stability, economy and generalizability. And when it achieves equilibrium it *quests*. (See Reference 4.) Shortly these children will be bored with Two Rubies and they'll want a go at Three Rubies. Watch this space. (See Note 2.)

The Genevan psychologist and researcher Hermina Sinclair said, "All of us concerned with education should view children as wearing signboards saying 'Under Construction'. No, wait a moment. I didn't say that strongly enough. All of us should look at *people* as wearing signboards saying, 'Under Construction—Self Employed'." (See Reference 5.)

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Notes

1. In fact, this answer is not correct. The 1 in C-3 could refer to a Ruby in D-2. Then, by the same sort of thinking given by the pupil, the second Ruby must be in A-4. One more bit of information, for example D-3, would resolve things.

2. In fact, children did tackle three Rubies with enthusiasm and success. (The teacher did not use a handheld because the existing version of Treasure Hunt had only one- and two-jewel games.) Here are the data, in order, from their first game:

B-2, 1   D-3, 2   C-2, 2   B-4, 2   C-4, 2   D-2, 2   D-1, 1   D-4, 1

Can you find the Rubies? Do you need more information?

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